1

Given:

$$U_1(y) = 1000y - 5y^2 \tag{1}$$
$$U_2(y) = 50000 + 200y \tag{2}$$

1.a

$$\max_y U_1(y) : 1000y - 5y^2 \tag{1}$$
$$\text{F.O.C.: } 1000 - 10y = 0 \tag{2}$$
$$y^* = 100 \tag{3}$$

1.b

$$\max_y U_1(y), U_2(y) : 1000y - 5y^2 + 50000 + 200y \tag{1}$$
$$1200y - 5y^2 + 50000 \tag{2}$$
$$\text{F.O.C.: } 1200 - 10y = 0 \tag{3}$$
$$y^{**} = 120 \tag{4}$$

1.c

In order to see if any negotiations will take place between the beekeeper and apple grower, we must first find each of their profits when the number of beehives is at the beehive keeper's maximum profit function and when the number of beehives is at both the beehive keeper and apple grower's maximum profit function, or in other words, when the number of beehives is at 100 and 120.

$$U_1(100) = 1000(100) - 5(100^2) = 100000 - 50000 = 50000 \tag{1}$$
$$U_1(120) = 1000(120) - 5(120^2) = 120000 - 72000 = 48000 \tag{2}$$
$$U_2(100) = 50000 + 200(100) = 50000 + 20000 = 70000 \tag{3}$$
$$U_2(120) = 50000 + 200(120) = 50000 + 24000 = 74000 \tag{4}$$

As we see, the profits gained at 120 beehives by the apple grower is larger than the loss in profit by the beehive keeper, therefore, there is room to negotiate for both parties. More specifically, with 120 beehives, the apple grower will earn \$4000 additional profit and the beehive keeper will lose \$2000 profit. The money the apple grower will pay will be no more than \$4000 to the beehive keeper and the beehive keeper will accept no less than \$2000 to maintain 120 beehives, assuming zero transactions cost.

1.d

$$\max_y \pi = 1000y - 5y^2 + s(y - 100) \tag{1}$$
$$\text{F.O.C.: } 1000 - 10y + s = 0 \tag{2}$$
$$s = 10(y) - 1000 \tag{3}$$
$$s = 10(120) - 1000 \tag{4}$$
$$s = 200 \tag{5}$$

To go from 100 to the socially optimal number of beehives (120), we found the total subsidy is \$200, therefore, the per unit beehive subsidy would be $200/120 = \$1.67$.

2

Given:

$$C_w = 0.01y^2 \tag{1}$$
$$C = 100 + 2y + 0.04y^2 \tag{2}$$

2.a

Without compensating for the additional cost of cleaning the polluted water, the profit function would simply be:

$$\pi = P_y y - C(y) \tag{1}$$
$$\max_y \pi : P_y y - C(y) \tag{2}$$
$$10y - 100 - 2y - 0.04y^2 \tag{3}$$
$$\text{F.O.C.: } 10 - 2 - 0.08y \tag{4}$$
$$y^* = 100 \tag{5}$$

2.b

Incorporating the additional cost of cleaning the polluted water, the profit function would be altered as:

$$\pi = P_y y - C(y) - C_w(y) \tag{1}$$
$$\max_y \pi : P_y y - C(y) - C_w(y) \tag{2}$$
$$10y - 100 - 2y - 0.04y^2 - 0.01y^2 \tag{3}$$
$$8y - 100 - 0.05y^2 \tag{4}$$
$$\text{F.O.C.: } 8 - 0.01y = 0 \tag{5}$$
$$y^{**} = 80 \tag{6}$$

2.c

$$\pi(80) = 10y - 100 - 2y - 0.04y^2 - ty \tag{1}$$
$$\text{F.O.C.: } 10 - 2 - 0.08y - t \tag{2}$$
$$t = 0.08y - 8 \tag{3}$$
$$= 0.08(80) - 8 \tag{4}$$
$$t^* = \$1.60 \tag{5}$$

2.d

To consider if there will be an agreement reached upon the residents and the farmers of the Clean Water Country, we must find the profit of the farmers under private and socially optimal the farmers' output:

$$\pi(100) = 10(100) - 2(100) - 0.04(100^2) = 300 \tag{1}$$
$$\pi(80) = 10(80) - 2(80) - 0.04(80^2) = 284 \tag{2}$$

Additionally, since the right to pollute is given to the farmers, the entire cost of cleaning the polluted falls on the residents, so we must find the cost of cleaning the water under the private and socially optimal the farmers' output:

$$C_w(100) = 0.01y^2 = 0.01(100^2) = 100 \tag{1}$$
$$C_w(80) = 0.01(80^2) = 64 \tag{2}$$

Therefore, to produce from the privately optimal output to the socially optimal output, farmers would lose $300 - 284 = \$16$ profit and the residents would incur $100 - 64 = \$36$ cost to clean up the polluted water. The farmers would accept no less than \$16 to produce 80 output and residents would pay up to \$36 for the farmers to produce 80 output. With zero transaction costs, we would see an agreement between the farmers and the residents for the residents to pay more than \$16 and less than \$36 to the farmers to reduce output from 100 to 80.

3

Given:

$$MB_H = 12 - y \quad (1)$$
$$MB_L = 6 - 0.5y \quad (2)$$
$$y = z \quad (3)$$

3.a

Assuming one price for every y, we know both the high and low-efficiency producers will continue to produce y until the marginal net benefits equal zero.

$$MB_H = 12 - y = 0 \quad (1)$$
$$y_H^* = 12 \quad (2)$$

$$MB_L = 6 - 0.5y = 0 \quad (1)$$
$$y_L^* = 12 \quad (2)$$

Therefore, the amount of pollutant emitted by each producer would be 12z , or 24z in total.

3.b

If the government imposes a regulation that requires each producer to cut their pollution by half, both the high and low-efficient producers will then produce 6 units of y each, and each producer would also emit 6 units of pollutant z as well.

3.c

$$MB_H = MB_L \quad (1)$$
$$12 - y_H = 6 - 0.5y_L \quad (2)$$
$$y_H + y_L = 12 \quad (3)$$
$$y_H = y_L - 12 \quad (4)$$
$$12 - (12 - y_L) = 6 - 0.5y_L \quad (5)$$
$$1.5y_L = 6 \quad (6)$$
$$y_L = 4 \quad (7)$$
$$y_H = 8 \quad (8)$$

Therefore, the high-efficient producer would produce 8 output and release 8z and the low-efficiency producer would produce 4 output and release 4z. The equilibrium price for the emission right would $MB_H = P = 12 - 8 = \$4$.

3.d

$$y = 0.5z \tag{1}$$
$$z_H + z_L = 12 \tag{2}$$
$$y_H + y_L = 24 \tag{3}$$
$$y_H = 24 - y_L \tag{4}$$
$$12 - (24 - y_L) = 6 - 0.5y_L \tag{5}$$
$$-12 + y_L = 6 - 0.5y_L \tag{6}$$
$$1.5y_L = 18 \tag{7}$$
$$y_L = 12 \tag{8}$$
$$y_H = 12 \tag{9}$$

Therefore, each producer would have 12 output and release the corresponding 6z. The equilibrium price for the emission right would be $MB_H = P = 12 - 12 = \$0$. This makes sense since due to the improvement in pollution technology, the restriction of pollutants is not binding and both the high and low-efficient would produce until their marginal net benefit is zero and they would not trade emission rights with each other at all.

3.e

Since producers invested in cleaner low-sulfur coal mines, the high-polluting sulfur coal mines have been out-competed by the cleaner western coal mines. This means power stations have been able to follow regulatory restrictions on sulfur emissions with technological improvements. In other words, because of technological improvement, the sulfur pollution restriction is less binding since the mines pollute less.